Exploring Exponential Models

- The general form of an exponential function is $y = ab^x$, where $a$ is the initial amount and $b$ is the growth or decay factor.
- To find $b$, use the formula $b = 1 + r$, where $r$ is the constant rate of growth or decay. If $r$ is a rate of growth, $r$ is positive. If $r$ is a rate of decay, $r$ is negative. Therefore, if $b$ is greater than 1, the function models exponential growth. If $b$ is between zero and 1, the function models exponential decay. When you see words like increase or appreciation, think growth. When you see words like decrease or depreciation, think decay.
- For an exponential function, the $y$-intercept is always equal to the value of $a$. $(0, a)$

Exponential Growth & Decay function: $y = ab^x$

Examples:

Determine whether the function represents exponential growth or exponential decay. Then find the $y$-intercept.

1. $y = 8000(1.15)^x$ growth
2. $y = 20(0.75)^x$ decay
3. $y = 15(\frac{1}{2})^x$ decay
4. $f(x) = 6\left(\frac{5}{2}\right)^x$ growth

Carl’s weight at 12 yr is 82 lb. Assume that his weight increases at a rate of 16% each year. Write an exponential function to model the increase. What is his weight after 5 years?

Step 1 Find $a$ and $b$.
- $a$ is the original amount.
- $b = 1 + 0.16 = 1.16$

Step 2 Write the exponential function that models the situation.

Use the formula.

Step 3 Calculate.

Substitute 5 for $x$.

Use a calculator.

Carl will weigh about 172.18 lbs in 5 years.

A motorcycle purchased for $9000 today will be worth 6% less each year. How much will the motorcycle be worth at the end of 5 years?

Step 1 Find $a$ and $b$.
$9000$  
$a$ is the original amount.  
$b = 1 + .06$  
$b = 1.06$  
Step 2 Write the exponential function.  
$y = a \cdot b^x$  
Use the formula.  
$y = 9000 (1.06)^x$  
Substitute.  
Step 3 Calculate.  
$y = 9000 (1.06)^5$  
Substitute 5 for $x$.  
Use a calculator.  
The motorcycle will be worth about $6605.14$ after 5 years.

Examples

Write an exponential function to model each situation. Find each amount after the specified time.

5. A tree 3 ft tall grows 8% each year. How tall will the tree be at the end of 14 yr? Round the answer to the nearest hundredth.

\[
a = 3 \\
b = 1 + .08 \\
r = .08 \\
x = 14 \\
y = 3 (1.08)^{14} \\
y \approx 8.81 	ext{ ft}
\]

6. The price of a new home is $126,000. The value of the home appreciates 2% each year. How much will the home be worth in 10 yr?

\[
a = 126,000 \\
r = .02 \\
x = 10 \\
y = 126000 (1.02)^{10} \\
y \approx 153,593.30
\]

7. A butterfly population is decreasing at a rate of 0.82% per year. There are currently about 100,000 butterflies in the population. How many butterflies will there be in the population in 250 years?

\[
a = 100,000 \\
r = - .0082 \\
b = 1 - .0082 \\
x = 250 \\
y = 100000 (1.9918)^{250} \\
y \approx 18,765
\]

8. A car depreciates 10% each year. If you bought this car today for $5000, how much will it be worth in 7 years?

\[
a = 5000 \\
x = 7 	ext{ years} \\
r = .10 \\
b = 1 - .1 = .9 \\
y = 5000 (.9)^7 \\
y \approx 2391.48
\]